Matrices

Introduction:

Cayley a French Mathematician; discovered Matrices in the year 1860. Matrices have great Utility in many branches of applied Mathematics; Machanics and Businen.

Definition:

A system of mn numbers arranged in a rectangular formation along m rows and n Columns and bounded by the brackets [ ] is called an m by n matrix; which is written as m x n matrix. A Matrix is also defined by a Single Capital letter.

$$\text{Thus } A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ -- & -- & & -- & & -- \\ a_{i1} & a_{i2} & & \dots a_{ij} & \dots & a_{in} \\ --- & --- & & -- & & -- \\ a_{m1} & a_{m2} & & a_{mj} & & a_{mn} \end{bmatrix}$$

is a matrix of order mn. It has m rows and n columns. Each of the mn numbers is called an element of the matrix.

To locate any particular element of a matrix, the elements are denoted by a letter followed by two suffixes which respectively specify the rows and the columns. Thus a_{ij} is the element in the ith row and j-th columns of A. In this notation, the matrix A is denoted by $[a_{ij}]$

Special Types of Matrices:

(1) Row and Column Matrices:

A Matrix having a single rows is called a row matrix

e.g.; $\begin{bmatrix} 1, & 3, & 5, & 7 \end{bmatrix}$

A matrix having a single column is called a column matrix; e.g.; $\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$

Row and Column matrix (Matrices) are some times called Row vectors and Coloumn vectors.

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(2) Square Matrix: A matrix having n rows and n columns is called a square matrix of order n.

The determinant having the same elements as the square matrix A is called the determinant of the matrix and is denoted by the symbol $|A|$.

For example; If

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

Then $$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

The diagonal of this matrix Conteining the elements 1, 3, 5 is Called the leading or; Principal diagonal. The Sum of the diagonal elements of a square matrix A is Called the trace of A.

A Square Matrix is said to be Singular; if its determinant is Zero Otherwise Non-Singular.

(3) Diagonal Matrix : A square Matrix all of whose elements except those in the leading diagonal, are Zero is called a diagonal Matrix

A diagonal Matrix whose all the leading diagonal elements are equal is called a Scalar Matrix. for example;

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 6 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

are the diagonal and Scalar Matrices respectively.

(4) Unit Matrix : A diagonal matrix of order n which has unity for all its diagonals ~~elem~~ ~~ent~~ elements, is called

a Unit matrix of Order n and is denoted by I_n. For example; Unit matrix of Order 3 ~~ia~~ is;

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(5) Null Matrix : If all the element of a Matrix are zero, it is called a Null Matrix or Zero matrix and is denoted by '0' e.g.;

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ is a null Matrix.

(6) Systematic and Skew- Systematic Matrices:

A Square matrix $A = [a_{ij}]$ is said to be systematic when

$a_{ij} = a_{ji}$ for all i and j.

If $a_{ij} = -a_{ji}$ for all i and j so that all the leading diagonal elements are zero, then the matrix is called a skew-symmetric Matrix. Examples of symmetric and skew-symmetric matrices are as given;

$$\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \text{ and } \begin{bmatrix} 0 & h & -g \\ -h & 0 & f \\ g & -f & 0 \end{bmatrix}$$

(7) Triangular Matrix :

A Square matrix all of whose elements below the leading diagonal are zero, is Called an upper triangular matrix.

A square matrix all of whose elements above the leading diagonal are zero, is called lower triangular matrix. Thus

$$\begin{bmatrix} a & h & g \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 1 & -5 & 4 \end{bmatrix}$$

are upper and lower triangular matrices respectively.

Equality of Matrices: Two matrices A and B are said to equal if and only if;

(i) They are of the same order;

and (ii) each element of A is equal to the corresponding element of B.

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